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IMS Data Fusion and the Possibilities of Dempster-Schafer Theory

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P3.5-476

POSTER

Health Canada Santé Canada

National Nuclear Security Administration

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under contract DE-AC52-07NA27344. Lawrence Livermore National Security, LLC. LLNL-PRES-822432

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ABSTRACT

The International Monitoring System (IMS) is comprised of multiple types of sensors that provide verification information. While each piece of information is useful for verification, the full benefit of multi-technology measurements has not been fully taken advantage of. Data Fusion is an approach that seeks to integrate disparate sources of data into a unified and comprehensive event analysis. Several approaches (e.g. cost-function analysis, Bayesian inference) have demonstrated the power and benefit of data fusion approaches for Treaty verification. However, an important problem in the data fusion process arises when not all information is consistent, or believable. Dempster-Schafer theory provides a statistical means to reconcile evidentiary beliefs in the data fusion process. This poster will describe how inconsistent evidence may arise within the IMS, and show how Dempster-Schafer theory can help to reconcile evidence in a data fusion process and support the event analysis process for National Data Centres.

This work was performed under the auspices of the U.S. Department of Energy by Lawrence Livermore National Laboratory under Contract DE-AC52-07NA27344. This abstract is LLNL-ABS-817217.

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INTRODUCTION

Dempster-Shafer Theory (DST) (theory of belief functions, evidence theory) is a tool that can be applied to problems involving uncertainty and conflicting evidence

- DST works with epistemic evidence (situations with a lack of system knowledge) or when information about parameters is conflicting or no probability distributions are suitable
- Encodes a degree of (subjective) belief for each piece of evidence and a framework to reconcile them
- In evidence theory there are two complementary measures of uncertainty: belief (lower uncertainty bound) and plausibility (upper uncertainty bound).

Two key concepts:

1. Obtain the belief of a desired question through the probability of an associated question
   - Sum of belief functions need not add to 1 (multivalued mapping, not a probability)
   - Does not require a significant statistical data to answer a question of interest
2. Provides a rule to combine degrees of belief from independent pieces of evidence
   - Evidence can either reinforce or erode confidence
   - Reasoning chain is always weaker than the weakest link (more conservative approach)

Can be naturally applied in Bayesian inference frameworks

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METHODS

Let S be the set of answers or *frame* for Q_1, with $P(s)$ the probabilities s of S and let T be the frame for $Q_{2,}$ which is the answer we seek

Given subset A of T, let $\Gamma(s)$ be a subset of T consisting of answers to Q_2 that are not ruled out by s.

Then,

s is somewhere in A when

$$\Gamma(s) \subseteq A \qquad (1)$$

The degree of belief in A, $bel(A)$ is the total probability of all s that satisfy (1),

$$bel(A) = P\{s|\Gamma(s) \subseteq A\}$$

With multiple pieces of evidence, s_1 and s_2, Γ becomes

$$\Gamma(s_1, s_2) = \Gamma_1(s_1) \cap \Gamma_2(s_2)$$

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METHODS

Let X be the universe of all possible sets for system, then

- 2^X is the power set of all possible subsets of X
- A belief mass, m, is assigned on $[0,1]$ (basic probability assignment) for all X such that:

$$m(\emptyset) = 0$$

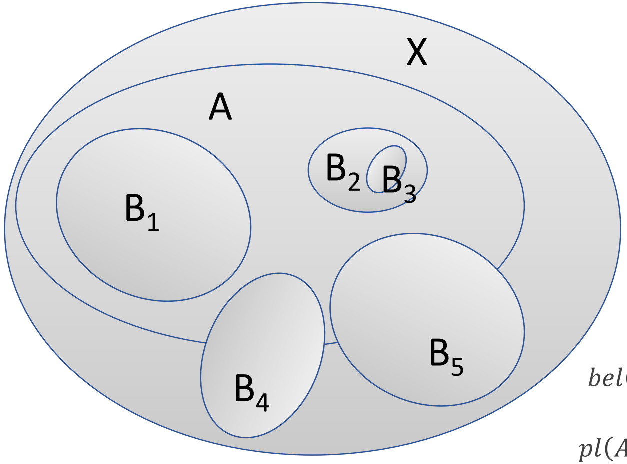
$$\sum_{A \in 2^X} m(A) = 1$$

Then $bel(A)$ is:

$$bel(A) = \sum_{B|B \subseteq A} m(B)$$

And

$$pl(A) = \sum_{B|B \cap A \neq \emptyset} m(B)$$



$$bel(A) = m(B_1) + m(B_2) + m(B_3)$$

$$pl(A) = m(B_1) + m(B_2) + m(B_3) + m(B_4) + m(B_5)$$

The above example shows how to calculate belief and plausibility for a simple case. Here X is the universe set and A is the considered subset of a system. Then the belief mass is distributed as shown for the conflicting sets B_1, B_2, B_3, B_4 and B_5.

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METHODS

In DST the belief and plausibility form upper and lower bounds of probability

$$\mathrm{Bel}(A) \leq \Pr(A) \leq \mathrm{pl}(A)$$

Given either mass, belief or plausibility, the others can be derived.

Combining independent evidence

$$m_{12}(A) = \frac{\sum_{B \cap C = A} m_1(B) m_2(C)}{1 - K}, \mathrm{A} \neq \emptyset$$

$$m_{12} \neq \emptyset$$

$$K = \sum_{B \cap C \neq \emptyset} m_1(B) m_2(C)$$

Caution: Sometimes normalizing factor K can lead to counterintuitive results.

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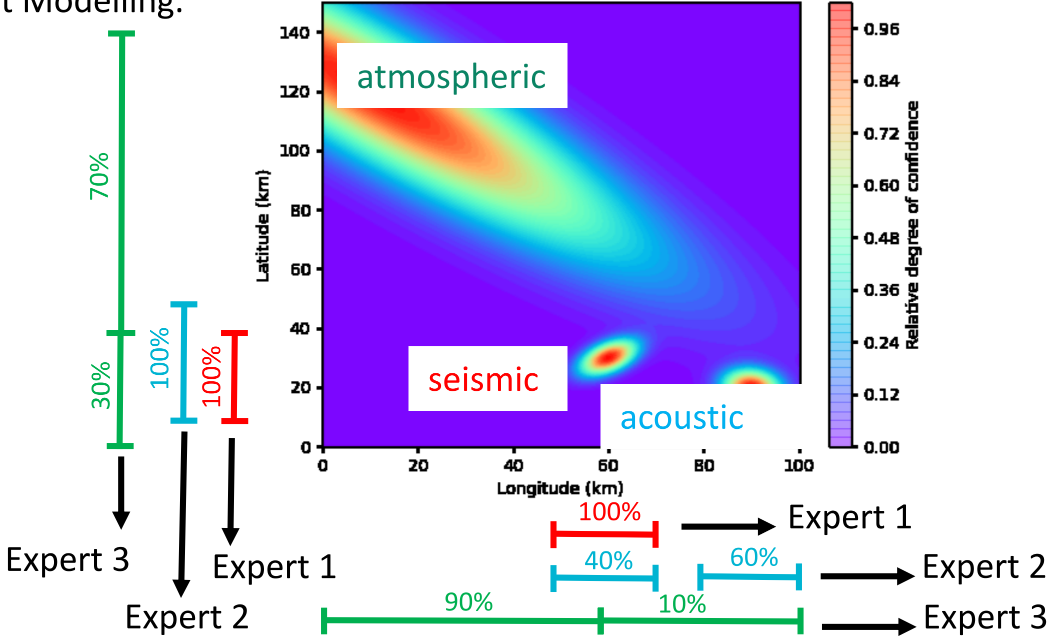
METHODS

Application to a CTBT Problem – Sensor Fusion Test with Synthetic Data

- The IMS has registered signals from seismic, hydro-acoustic, and radionuclide
- The radionuclide detect e location using Atmospheric Transport Modelling.



Results based in various technologies may be inconsistent

Experts provide intervals for each result

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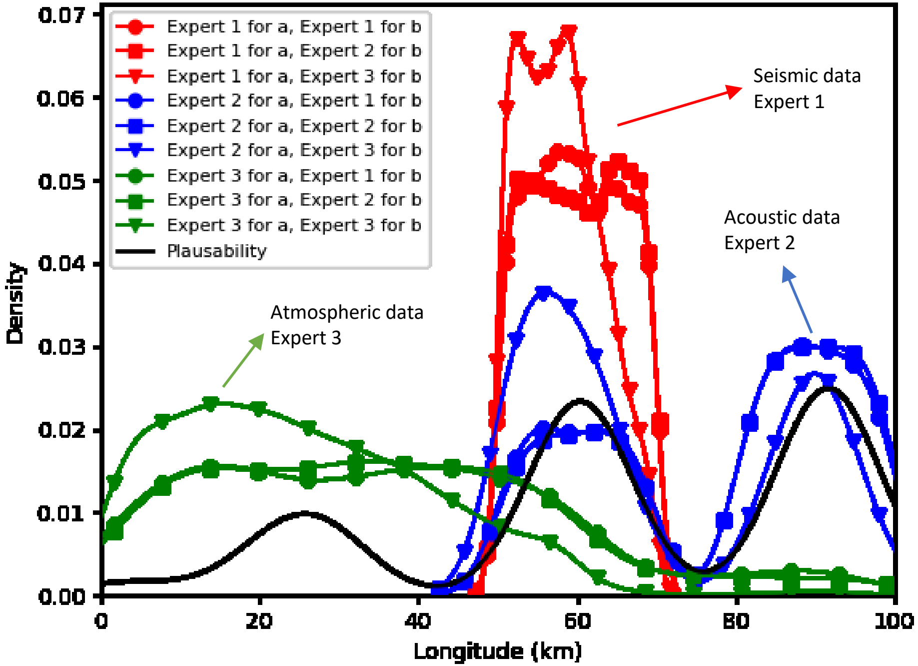
RESULTS

- **Probability theory** results are conflicting due to combinations of different experts
  - Expert 1 only considers seismic technology
  - Expert 2 considers seismic and acoustic technology
  - Expert 3 mainly considers atmospheric technology
- **DST** plausibility **fuses** the conflicting evidence from all experts and technology in a **single** assessment



Probability density functions for longitude. The figure includes probability theory results (red, blue and green lines) and DST results (black line)

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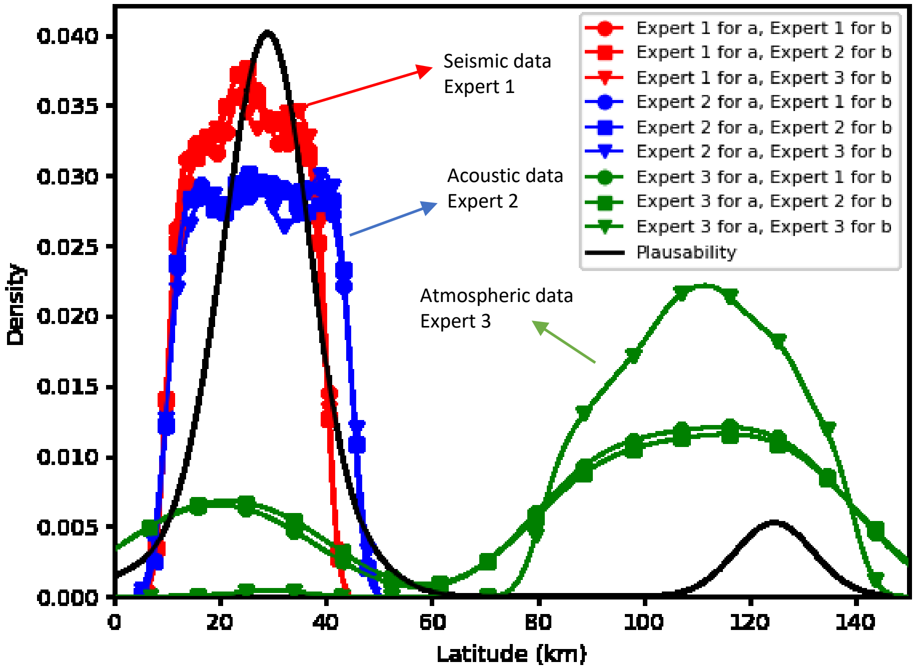

Probability density functions for latitude. The figure includes probability theory results (red, blue and green lines) and DST results (black line)

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CONCLUSIONS

- DST is often applied to sensor fusion as a means of reconciling measurements
  - For CTBT, it can reconcile multiple event locations from SHI, ATM/RN analyses
  - DST can be used at National Data Centres to aid in verification, for example, given an event with no ground truth or definitive information is known:
    - Knowledge of historical observations can inform the subjective interval assignment when analyzing an event of interest (imprecise probabilities)
  - Provides a more conservative estimate than standard probabilities through the incorporation of epistemic uncertainty
  - The sensor fusion synthetic test demonstrates that DST can successfully fuse data from different technologies and different expert assessments using the technologies
- Application of DST Challenges
  - Assignment of mass function is subjective
  - Requires expertise to avoid non-intuitive outcomes

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We use Dakota (https://dakota.sandia.gov/) to compute DST results. The authors would like to acknowledge the help from the Dakota team (Adam Stephens and Laura Painton)

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