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The Optimised Local Renyi Entropy-Based Shrinkage Algorithm for Sparse TFD Reconstruction

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POSTER

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ABSTRACT

- Time-frequency distributions (TFDs) are useful tools for non-stationary signals analysis. However, due to the presence of unwanted cross-terms useful information extraction from TFDs has proven to be a challenging task, especially in the case of noise-corrupted real-life signals. One way to suppress the cross-terms is by employing compressive sensing methods that enforce sparsity in the resulting TFD.
- In this work, we have developed a sparse reconstruction algorithm that reconstructs a TFD from a small sub-set of signal samples in the ambiguity domain. The algorithm utilises the information from both the short-term and the narrow-band Rényi time-frequency entropies, while its parameters are optimised using evolutionary meta-heuristic methods.
- Results are presented for both synthetic and real-life signals in noise, and compared to the state-of-the-art sparse reconstruction algorithms.

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INTRODUCTION

Time-Frequency Signal Analysis

- The Wigner-Ville Distribution (WVD) is the most commonly used method for TFD calculation defined as

$$W_z(t,f)=\int_{-\infty}^{\infty} z\Big(t+\frac{\tau}{2}\Big)z^*\Big(t-\frac{\tau}{2}\Big)e^{-j2\pi f\tau}d\tau, \quad (1)$$

which introduces wanted components (auto-terms) and highly oscillatory unwanted components (cross-terms).

- The cross-terms can be suppressed in the WVD post-processing by applying a low-pass filter to the ambiguity function (AF):

$$A_z(\nu,\tau)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} W_z(t,f)e^{j2\pi(f\tau-\nu t)}dtdf, \quad (2)$$

which leaves the auto-terms positioned at the AF origin and filters out the cross-terms positioned through the rest of the domain:

$$\mathcal{A}_z(\nu,\tau)=g(\nu,\tau)A_z(\nu,\tau), \quad (3)$$

where $g(\nu,\tau)$ is the AF filter kernel.

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The Compressive Sensing (CS) Methods

- The cross-term suppression can be achieved with the sparse reconstruction. Eq. (3) can be rewritten in the matrix form:

$$\boldsymbol{\vartheta}_z(t,f) = \boldsymbol{\psi}^H \cdot \mathbf{A}'_z(\nu,\tau), \tag{4}$$

where $\boldsymbol{\vartheta}_z(t,f)$ is the sparse TFD, or the solution matrix, ψ^H is the Hermitian transpose of the domain transformation matrix representing the 2D Fourier transform equivalent to (2), and $\mathbf{A}'_z(\nu,\tau)$ is the CS-AF, or the observation matrix, which is a $N'_\tau \times N'_\nu$ rectangle containing the AF samples belonging to the auto-terms.

- The rest of the AF is calculated in a way which produces the sparsest TFD. This is an optimization problem with the ℓ_0-norm-based regularization function:

$$\begin{aligned} \boldsymbol{\vartheta}_z^{\ell_0}(t,f) &= \arg\min_{\boldsymbol{\vartheta}_z(t,f)} ||\boldsymbol{\vartheta}_z(t,f)||_0, \\ \text{subject to: } & ||\boldsymbol{\vartheta}_z(t,f) - \boldsymbol{\psi}^H \mathbf{A}'_z(\nu,\tau)||_2^2 \leq \epsilon, \end{aligned} \tag{5}$$

where ϵ is a user-defined solution tolerance.

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METHODS

The Local Rényi Entropies

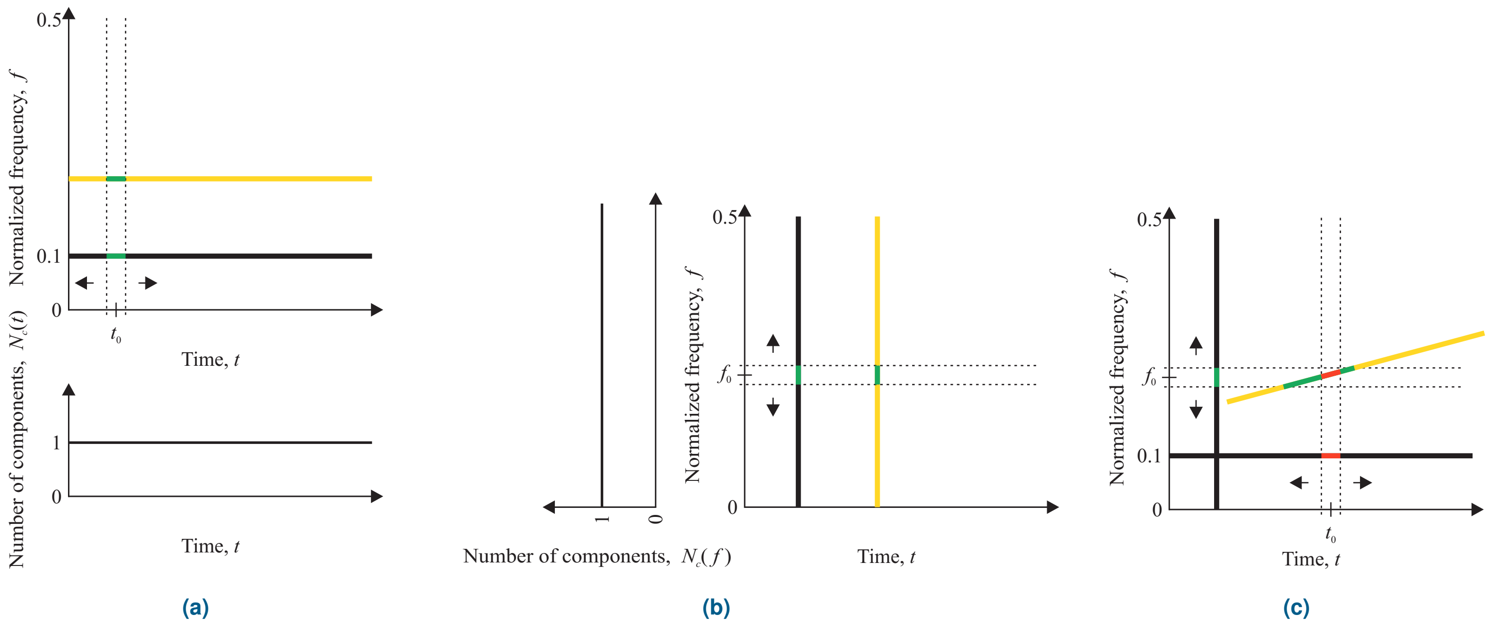

Figure 1: (a) Short-term Rényi entropy; (b) narrow-band Rényi entropy; (c) both local entropies on one-component signal.

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The Local Rényi Entropy Based Shrinkage Algorithm for Sparse TFD Reconstruction

- The proposed shrinkage algorithm is based on the Two-step iterative shrinkage/thresholding (TwIST) algorithm:

$$\left[\boldsymbol{\vartheta}_z^{\ell_0}(t,f)\right]^{[n+1]} = (1-\alpha)\left[\boldsymbol{\vartheta}_z^{\ell_0}(t,f)\right]^{[n-1]} + (\alpha-\beta)\left[\boldsymbol{\vartheta}_z^{\ell_0}(t,f)\right]^{[n]} + \beta \cdot \text{shrink}\Big\{\left[\boldsymbol{\vartheta}_z^{\ell_0}(t,f)\right]^{[n]} + \boldsymbol{\psi}^H\Big(\mathbf{A}_z^{'}(\nu,\tau) - \boldsymbol{\psi}\left[\boldsymbol{\vartheta}_z^{\ell_0}(t,f)\right]^{[n]}\Big)\Big\}, \tag{6}$$

  where α and β are user-defined TwIST relaxation parameters.
- *shrink*$\{\cdot\}$ operator is based on the **short-term** and the **narrow-band Rényi entropies** which give information on the number of signal components in each time- or frequency-slice, $N_c(t)$ or $N_c(f)$, respectively.

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- CS-AF filtering concentrates mainly on the auto-terms; hence, the obtained sparse TFD has auto-terms with larger non-negative energy surface than cross-terms.
- The shrinkage algorithm leaves samples belonging to $N_c(t)$ or $N_c(f)$ largest surfaces in time- or frequency-slice.
- Parameters δ_t/δ_f control the number of samples left in the final time-/frequency-slice.
- The algorithm performance is controlled by the percentage of utilization of each Rényi entropy information, controlled by the parameter p:

$$\varsigma_z(t,f) = p \cdot \varsigma_z^t(t,f) + (1-p) \cdot \varsigma_z^f(t,f), \tag{7}$$

where $\varsigma_z^t(t,f)$ and $\varsigma_z^f(t,f)$ are TFDs obtained by the proposed shrinkage performed over time- or frequency-slices, respectively.

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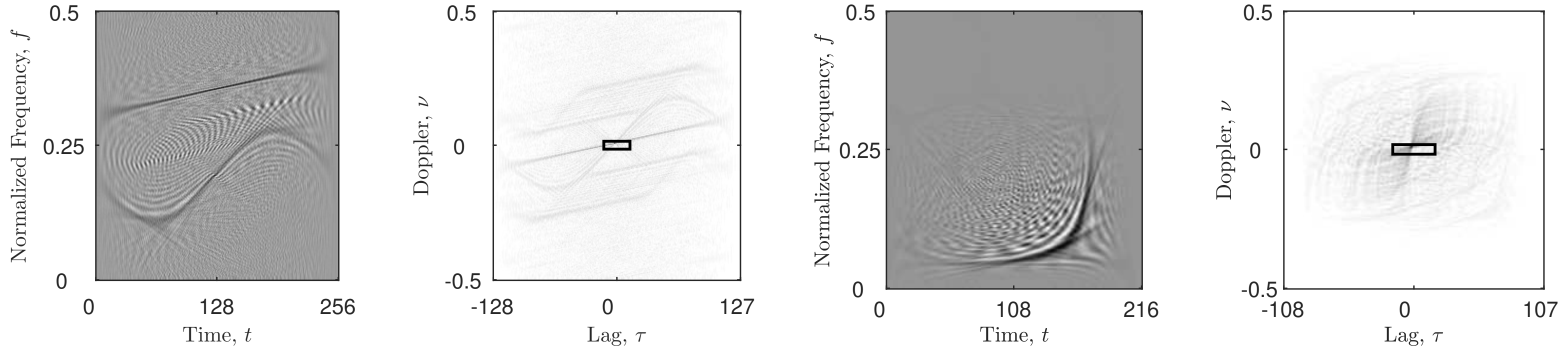
RESULTS

Considered Test Signals



(a) (b) (c) (d)

Figure 2: WVD and its respective AF of: (a),(b) z_{s}; (c),(d) z_{r}. The automatically selected CS-AF area has been marked by a rectangle.

- z_{s} - synthetic signal composed of linear and sinusoidal FM components embedded in additive white Gaussian noise with signal-to-noise ratio = 3 dB.
- z_{r} - real-life gravitational signal (https://losc.ligo.org)
- The reconstruction performance has been compared to the following state-of-the-art reconstruction algorithms: TwIST, Sparse reconstruction by separable approximation (SpaRSA) and Split augmented Lagrangian shrinkage algorithm (SALSA).

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RESULTS

Parameters Optimization

- We have used the multi-objective optimization method based on the Particle swarm optimization (MOPSO) method; a stochastic optimization algorithm inspired by nature and social behaviour between birds in swarms.

Objectives which need to be minimized:

- mean squared errors between the local number of components (obtained by the short-term and the narrow-band Rényi entropy) in the starting and reconstructed TFDs, $\text{MSE}_{t,f}$ - preserve components resolution and consistency
- the number of regions with continuously-connected AF samples, N_r - preserves components connectivity

For the proposed algorithm, a multi-objective problem is formalized as:

$$\begin{aligned} \min\{\text{MSE}_t, \text{MSE}_f, N_r(\alpha, \beta, p, \delta_t, \delta_f)\}, \\ \text{s.t. } \alpha, p, \delta_t, \delta_f \in [0,1], \beta \in [0, 2\alpha]. \end{aligned} \tag{8}$$

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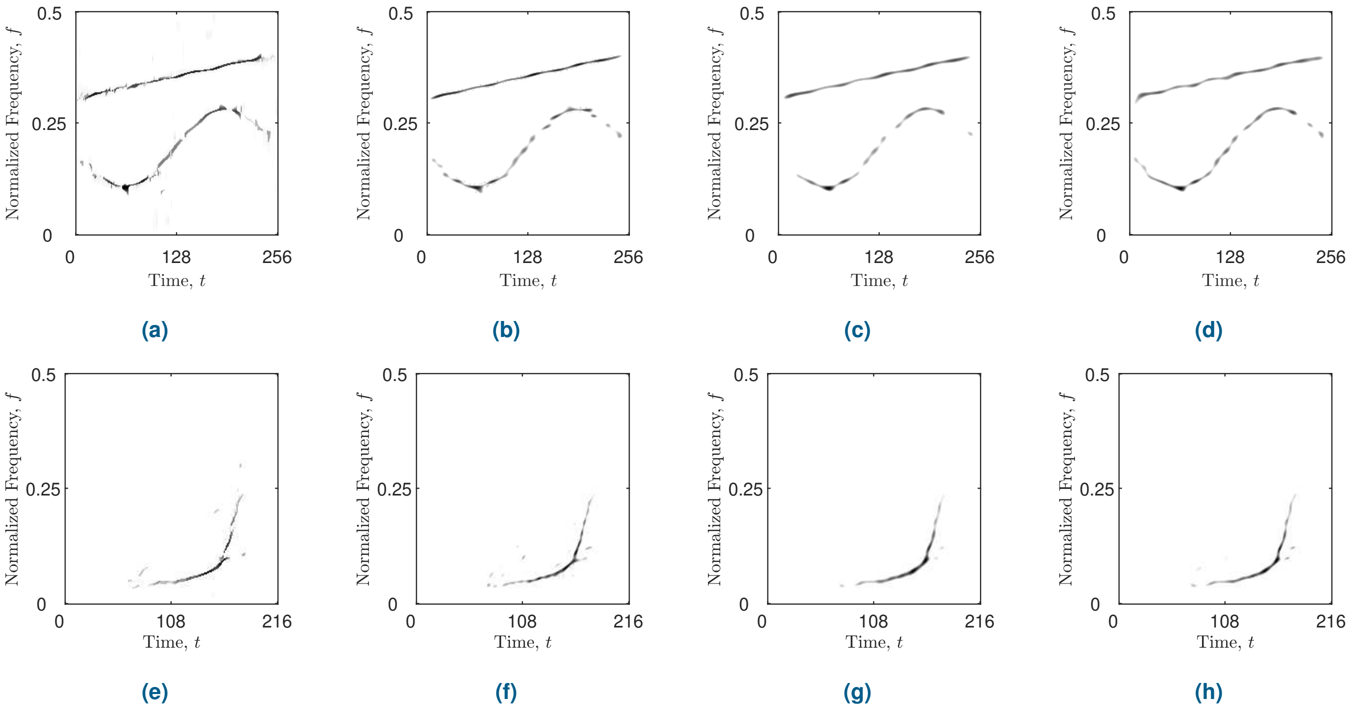

Figure 3: Reconstructed sparse TFDs of: (a) z_{s} with the proposed algorithm; (b) z_{s} with the TwIST algorithm; (c) z_{s} with the SpaRSA algorithm; (d) z_{s} with the SALSA algorithm; (e) z_{r} with the proposed algorithm; (f) z_{r} with the TwIST algorithm; (g) z_{r} with the SpaRSA algorithm; (h) z_{r} with the SALSA algorithm.

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RESULTS

Table 1: Comparison with the state-of-the-art algorithms. The bold values indicate the best performing/fastest reconstruction algorithm.

	Rényi		TwIST		SpaRSA		SALSA	
	$p = 0.816$ $\delta_t = 0.010$ $\delta_t = 0.948$	$p = 1$ $\delta_t = 0.913$ $\delta_t = 0.823$						
	z_s	z_r	z_s	z_r	z_s	z_r	z_s	z_r
MSE_t	0.0170	**0.0052**	**0.0132**	0.0194	0.0204	0.0339	0.0211	0.0218
MSE_f	**0.0110**	**0.0048**	0.0423	0.0074	0.0396	0.0111	0.0205	0.0077
N_r	17	11	18	21	6	**4**	**5**	7
$t[s]$	0.165	0.381	**0.191**	0.25	**0.138**	**0.112**	0.604	0.232

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CONCLUSIONS

- By utilizing both local Rényi entropies simultaneously, the proposed algorithm reduces inaccuracies of each entropy when analysing signals with components having different FM modulations.
- The proposed algorithm achieves competitive results when compared to the state-of-the-art sparse reconstruction algorithms, providing the best compromise between the objective functions and the algorithm execution time.

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