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A theoretical formulation of a 3D acoustic propagation model for stratified oceanic media based on an indirect BEM approach

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Abstract

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ABSTRACT

- This **work in progress** presents the first steps in the modelling of **3D** acoustic propagation in the ocean with Boundary Element Methods (**BEM**).
- The main motivation is building alternative methods to the well-known ones in the area (parabolic equation, ray tracing, normal modes, etc) taking advantage of BEM's intrinsic strengths.
- The theoretical foundations were built and the first results, verifications and evaluations on synthetic scenarios, are encouraging.
- Future steps include iterative solution of the main matricial system and accelerated evaluation of integrals.

Introduction

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INTRODUCTION

- Acoustic propagation modelling in the ocean is a key component to analize and understand received signals at hydroacoustic stations.
- Modelling acoustic 3D propagation for stratified ocean represents a challenge from the theoretical and computational point of view.
- Approaches based on Parabolic Equation, Normal Modes, Ray-tracing, among others constitute the *state of the art*.
- In recent years, the Boundary Element Method (BEM) has been applied to 3D propagation modelling for shallow waters [Li et al, 2019].

Pros and cons of BEM

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INTRODUCTION

Advantages of BEM

- In a 3D domain only discretization over its 2D boundary is required.
- The formulation automatically satisfies the Sommerfeld radiation condition so that no artificial boundary layers are required.
- Arbitrary bathymetry can be easily modelled through meshes.
- In comparison with marching solutions as one-way Parabolic Equation, BEM-based aproaches do not neglect backscattering effect.

Disadvantages

- Computationally expensive in time and memory storage.
- Up to now, only for shallow waters. Considering realistic sound speed profiles leads to include too many layers with the consequent computational limitations.

Theoretical formulation (1)

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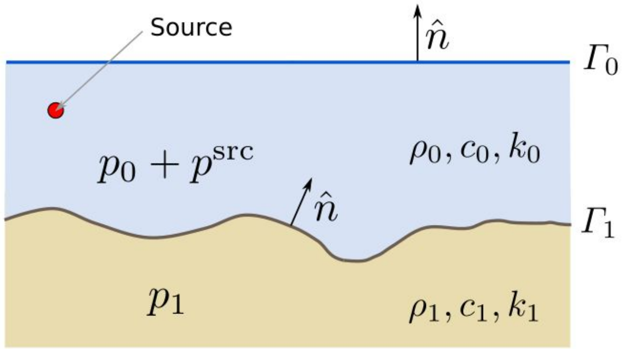
METHODS

- An harmonic source of frequency f generates a field p^{src} that, when interacting with the boundaries $\Gamma_i, (i = 0, 1)$ gives rise to pressure fields; p_0 in water and p_1 in marine sediments.



- Acoustic properties of each layer are density ρ_i, sound speed c_i, and wavenumber $k_i = 2\pi f / c_i$.

- To calculate p_0 and p_1, a Helmholtz equation

$$\nabla^2 p_i + k_i^2 p_i = 0$$

must be solved in each layer i subject to boundary conditions in Γ_1:

$$p_0 + p^{\mathrm{src}} = p_1,$$

$$\frac{1}{\rho_0}\partial_n(p_0 + p^{\mathrm{src}}) = \frac{1}{\rho_1}\partial_n p_1,$$

and boundary conditions in Γ_0:

$$p_0 + p^{\mathrm{src}} = 0.$$

Theoretical formulation (2)

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METHODS

- Using the Green Representation Formula the fields can be expressed with operators K, S (surface integrals)

$$K_i[\psi](x) = \int_\Gamma \frac{\partial G(k_i; x, y)}{\partial n_y} \psi(y)\, dS_y$$

$$S_i[\phi](x) = \int_\Gamma G(k_i; x, y)\, \phi(y)\, dS_y$$

applied on complex functions ψ and ϕ defined on Γ (indirect formulation). The auxiliary two-point function $G(k_i; x, y)$ is tipically the Green Free Space function.

- Using a Half-Space Green Function

$$G'(k_i; x, y) = \frac{e^{ik_i|x-y|}}{|x-y|} - \frac{e^{ik_i|x-y'|}}{|x-y'|},$$

allows to model the surface of the ocean as an infinite pressure release surface in an exact form so that integration over the boundary Γ_0 is no more needed. The y' stands for the orthogonal reflection of y regarding to Γ_0.

- Each pressure is written as a combination of K, S

$$\begin{cases} p_0(x) = K_0[\psi](x) + S_0[\phi](x) - \widetilde{p^{src}}(x) \\ p_1(x) = K_1[\psi](x) + S_1[\phi](x) \end{cases},$$

where $\widetilde{p^{src}}(x) = p^{src}(x')$

- The functions ϕ, ψ (called densities) are devoid of physical meaning and they represent the unknowns of the formulation.
- Applying the boundary conditions a system of Boundary Integral Equations (BIE) is obtained,

$$(\mathcal{A} + I)\begin{pmatrix} \psi \\ \phi \end{pmatrix} = g,$$

where $\mathcal{A}$ is a compact operator which assures uniqueness and stability. This will give a well behaved formulation when the problem is numerically solved. The data p^{src} and its derivative are contained in g.

Numerical method

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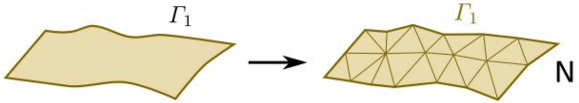
METHODS

- To solve the system a discretization procedure is carried out: from Γ_1 a triangular mesh of N elements is built



- Following a Collocation step on the BIE system, a discrete $2N \times 2N$ matricial system is obtained

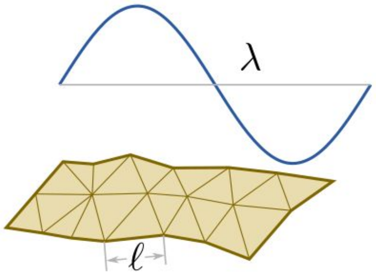
$$(\boldsymbol{A} + \boldsymbol{I}) \begin{pmatrix} \boldsymbol{\psi} \\ \boldsymbol{\phi} \end{pmatrix} = \boldsymbol{g},$$

where each element A_{ij} is an integral of the type

$$A_{ij} \sim \int_{\triangle_j} \mathrm{Ker}(x, y)\, dS_y$$

being $\mathrm{Ker}(x, y)$ the Green function or its normal derivative. Since the $\mathrm{Ker}(x, y)$ is singular when $x = y$ special integration techniques must be employed.

- An appropriate solution to the problem with BEM requires that the length ℓ of the triangle' edges in the mesh be lower enough in comparison to the wavelength. A criterion is $\ell < \frac{\lambda}{6}$.



- For high frequency or long range distances N increases leading to a very large $(2N)^2$ matricial system which can be a constraint in computational resources.
- This approach becomes **feasible when iterative solvers and acceleration algorithms are used** [Li et. al, 2019]

Relevant considerations

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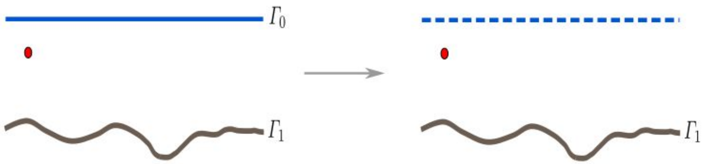
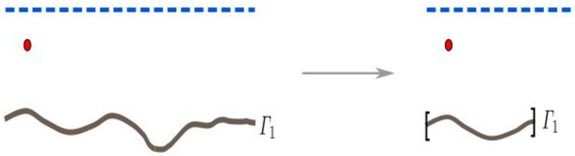
METHODS

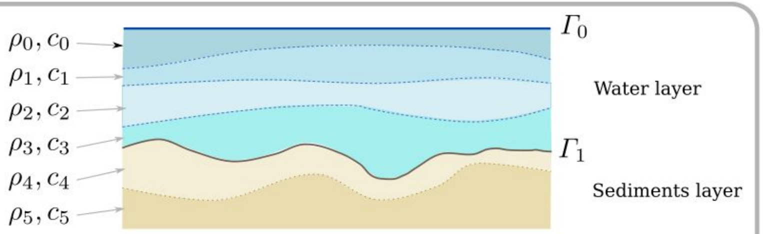


A minimal propagation problem would required meshes for boundaries Γ_0 and Γ_1. The formulation based on the Half-Space Green function presented here automatically satisfies boundary conditions on Γ_0, hence, only a mesh for Γ_1 is required.



The boundaries in real applications are almost infinite in extension, but the computational domain is bounded in order to lower the size of the matrix system.

This approach can be extended to one with multiple layers, where each mesh corresponds to a separation interface between isovelocity regions.



A real-life scenario

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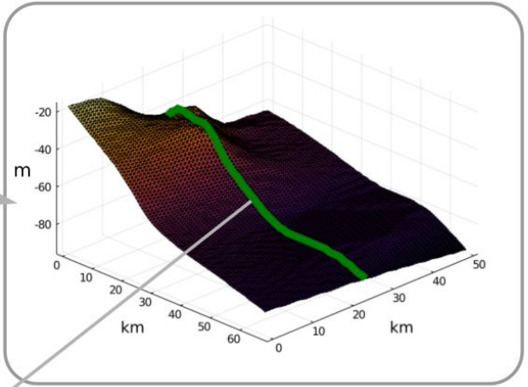
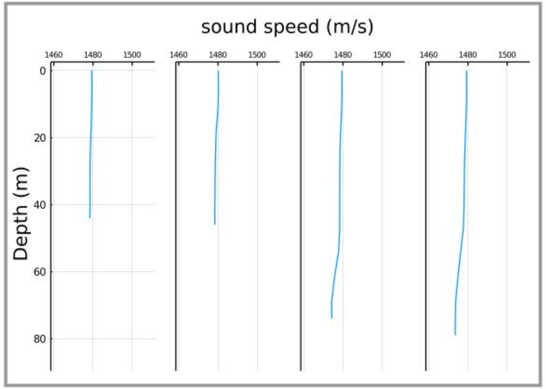
METHODS





As an example of a shallow water scenario, a rectangle inside the Argentinian Sea is shown with its bathymetry and representative sound speed profiles along the path indicated.

This region is a clear candidate for application of the BEM model.

Verification

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RESULTS

The BEM model is compared with a benchmark solution [Luo, 2016] for a Pekeris waveguide 100 m depth at 5 Hz.

The parameters used were:

$$\rho_0 = 1000 \text{ kg/m}^3 \quad c_1 = 1500 \text{ m/s}$$

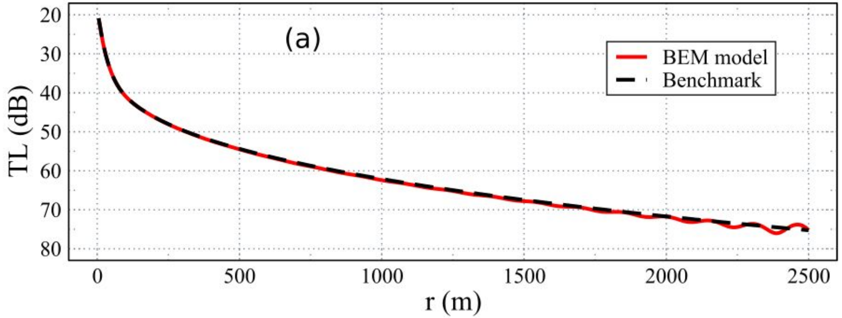
$$\rho_1 = 1800 \text{ kg/m}^3 \quad c_1 = 1800 \text{ m/s}$$

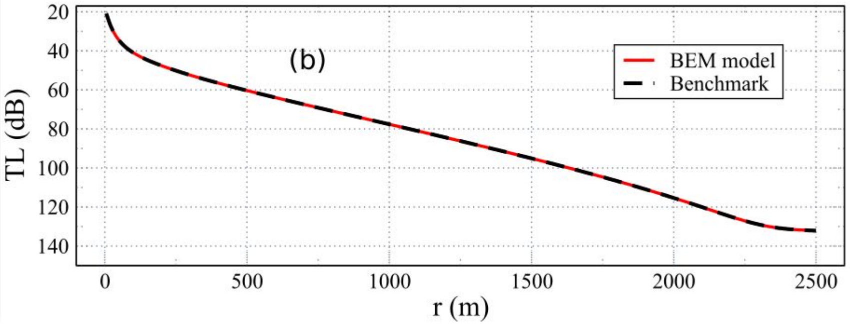
$$z_{\text{src}} = 36 \text{ m} \quad z_{\text{field}} = 46 \text{ m}$$

TL are evaluated along the line r = 0 m to r = 2500 m for $z = z_{\text{field}}$.

The mesh employed is a 2500 m radius disc with N= 50000 triangles.

Two cases are considered: (a) without attenuation (b) with attenuation given by $\alpha = 5$ dB / λ.





Results

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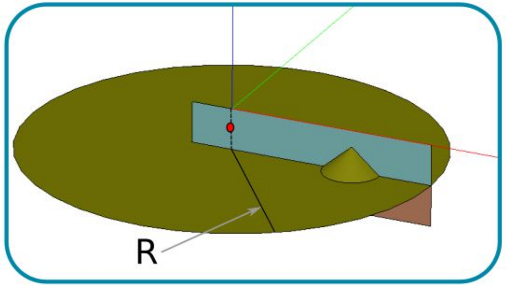
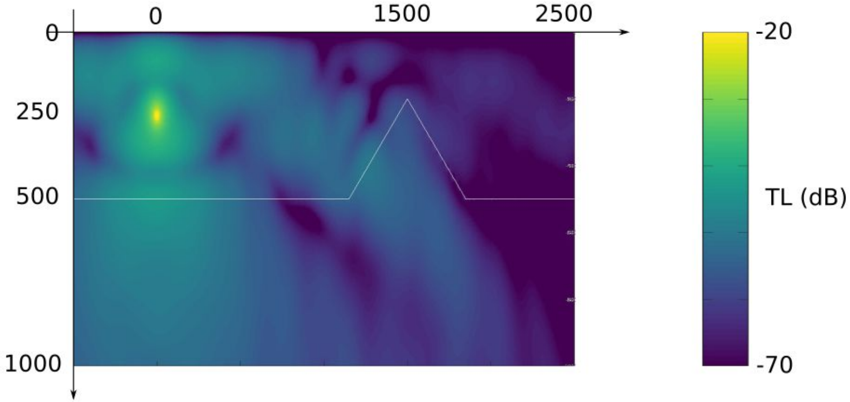
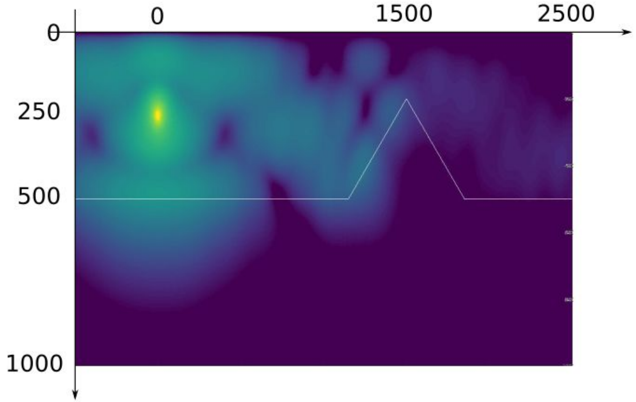
RESULTS

As an application of the model the TL for synthetic scenario consisting in a water layer 500 m depth with a conical mountain (300 m height) within is carried out.
TL is evaluated for the entire plane showed in two cases: (a) without attenuation (b) with attenuation $\alpha = 20$ dB / λ.
The source is located in z=250 m and r=0. The mesh used was a R = 2500 m radius disc with N= 50000 triangles.
The colorbar values were saturated for better contrast.







Conclusions

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CONCLUSIONS

- **BEM can provide a rigorous solution to the 3D propagation problem in an homogeneous layer**. For a stratified medium, which can be modelled as a multi-domain problem, BEM still provides a rigorous solution but the associated matrix system may become prohibitively large.
- Due to the memory storage and computational power requirements, **modelling large scenarios is very expensive**. Currently, in order to solve this issue, acceleration mechanisms based on convolution-FFT are under study.
- The use of the **Green Half-Space Function** in the theoretical formulation presented here is particularly suitable. This avoids generating a mesh of the ocean-air interface, taking advantage of limited computational resources.